

NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

REPORT No. 896

PRESSURE-SENSITIVE SYSTEM FOR  
GAS-TEMPERATURE CONTROL

By RICHARD S. CESARO and NORMAN MATZ



1948

## AERONAUTIC SYMBOLS

### I. FUNDAMENTAL AND DERIVED UNITS

Symbol	Metric			English	
	Unit	Abbreviation	Unit	Abbreviation	
Length----- Time----- Force-----	$l$ $t$ $F$	meter----- second----- weight of 1 kilogram-----	m s kg	foot (or mile)----- second (or hour)----- weight of 1 pound-----	ft (or mi) sec (or hr) lb
Power----- Speed-----	$P$ $V$	horsepower (metric)----- kilometers per hour----- meters per second-----	kph mps	horsepower----- miles per hour----- feet per second-----	hp mph fps

### 2. GENERAL SYMBOLS

$W$	Weight = $mg$	$\nu$	Kinematic viscosity
$g$	Standard acceleration of gravity = $9.80665 \text{ m/s}^2$ or $32.1740 \text{ ft/sec}^2$	$\rho$	Density (mass per unit volume)
$m$	Mass = $\frac{W}{g}$		Standard density of dry air, $0.12497 \text{ kg-m}^{-3}\text{-s}^2$ at $15^\circ \text{ C}$ and 760 mm; or $0.002378 \text{ lb-ft}^{-4}\text{-sec}^2$
$I$	Moment of inertia = $mk^2$ . (Indicate axis of radius of gyration $k$ by proper subscript.)		Specific weight of "standard" air, $1.2255 \text{ kg/m}^3$ or $0.07651 \text{ lb/cu ft}$
$\mu$	Coefficient of viscosity		

### 3. AERODYNAMIC SYMBOLS

$S$	Area	$i_w$	Angle of setting of wings (relative to thrust line)
$S_w$	Area of wing	$i_t$	Angle of stabilizer setting (relative to thrust line)
$G$	Gap	$Q$	Resultant moment
$b$	Span	$\Omega$	Resultant angular velocity
$c$	Chord	$R$	Reynolds number, $\rho \frac{Vl}{\mu}$ where $l$ is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at $15^\circ \text{ C}$ , the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)
$A$	Aspect ratio, $\frac{b^2}{S}$	$\alpha$	Angle of attack
$V$	True air speed	$\epsilon$	Angle of downwash
$q$	Dynamic pressure, $\frac{1}{2}\rho V^2$	$\alpha_0$	Angle of attack, infinite aspect ratio
$L$	Lift, absolute coefficient $C_L = \frac{L}{qS}$	$\alpha_i$	Angle of attack, induced
$D$	Drag, absolute coefficient $C_D = \frac{D}{qS}$	$\alpha_a$	Angle of attack, absolute (measured from zero-lift position)
$D_0$	Profile drag, absolute coefficient $C_{D_0} = \frac{D_0}{qS}$	$\gamma$	Flight-path angle
$D_i$	Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$		
$D_p$	Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$		
$C$	Cross-wind force, absolute coefficient $C_c = \frac{C}{qS}$		

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**By RICHARD S. CESARO and NORMAN MATZ**

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Cleveland, Ohio**

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# REPORT No. 896

## PRESSURE-SENSITIVE SYSTEM FOR GAS-TEMPERATURE CONTROL

By RICHARD S. CESARO and NORMAN MATZ

### SUMMARY

A thermodynamic relation is derived and simplified for use as a temperature-limiting control equation involving measurement of gas temperature before combustion and gas pressures before and after combustion. For critical flow in the turbine nozzles of gas-turbine engines, the control equation is further simplified to require only measurements upstream of the burner. Hypothetical control systems are discussed to illustrate application of the control equations.

### INTRODUCTION

The problem of gas-temperature control is becoming increasingly important for high-temperature applications, such as the gas-turbine power plant for aircraft. The acceleration of conventional gas-turbine power plants is currently induced by increasing the fuel flow to the engine. Because fuel flow is increased before the air flow increases, such a process of acceleration is accompanied by an increase in combustion-gas temperature to values that may exceed the temperature limitation of the engine. Temperature-limiting control is therefore required in order to eliminate the possibility of engine failure from overheating during this transient phase of operation.

The temperature limitation of an engine is determined by the temperature at which any engine component will fail. Because of the poor response characteristics inherent in a system of temperature control dependent on the sensing of metal temperatures, such a method is considered impracticable. The sensing and the control of combustion-gas temperatures appear to be more advantageous than an attempt to control directly the temperature of the critical engine component.

Limiting control of gas temperatures presents a difficult problem in that direct means of sensing combustion-gas temperatures of the magnitude encountered in gas-turbine engines are as yet unsatisfactory. The usual means of sensing temperature, such as thermocouples and resistance wires, are subject to errors resulting from radiation, conduction, and oxidation. In addition to these errors, the life of such instruments is comparatively short at the high temperatures to which they would be subjected. The thermodynamic methods presented herein were conceived to circumvent such difficulties.

Any means of sensing temperature that is selected for control purposes should be accurate, respond quickly, and pro-

vide a response that is easily incorporated into a control system. Measurement and interpretation of the thermodynamic changes that occur in the working fluid during the combustion process offer possibilities for meeting these requirements and obtaining temperature indications that can be used for control application. Temperature changes of the combustion gases are accompanied by pressure changes that are in effect instantaneous, are unaffected by errors of radiation and conduction, and can be utilized in a control system without modification or amplification.

Investigations have been made correlating temperature rise across the combustion chamber with pressure changes. These correlations, however, depend on combustion-chamber pressure losses requiring a calibration of the particular combustion chamber and the method appears impractical for control purposes.

A theoretical equation based on the thermodynamic properties of the combustion gas that correlates gas-temperature ratio with pressures upstream and downstream of the combustion zone was developed at the NACA Cleveland laboratory during 1947 and is presented herein. Means of simplifying this equation for control purposes are discussed and hypothetical control systems are presented to illustrate application of the control equations.

### DERIVATION OF CONTROL EQUATIONS

#### GENERAL CONTROL EQUATION

Determination of the final temperature of the gas from the thermodynamic relations involved is possible without direct sensing of the temperature in the hot gas stream by equating the mass flow of gas in a duct upstream of the point of addition of heat (station 1) to the mass flow downstream of the point of addition of heat (station 2). (The symbols used herein are defined in appendix A.)

The flow at station 2 equals the air flow at station 1 plus the fuel flow  $f$ , that is,

$$a_2 = a_1 + f$$

$$a_2 = \left(1 + \frac{f}{a_1}\right) a_1$$

When the equivalent gas-flow equations are substituted for  $a_1$  and  $a_2$ ,

$$E_2 A_2 \phi_2 \sqrt{2g\rho_2 \Delta P_2} = \left(1 + \frac{f}{a_1}\right) E_1 A_1 \phi_1 \sqrt{2g\rho_1 \Delta P_1} \quad (1)$$

The density  $\rho$  may be replaced by its equivalent  $\frac{p}{RT}$  and  $\phi$  by its corresponding relation  $\phi'$ , as shown in appendix B:

$$E_2 A_2 \phi'^2 \sqrt{\frac{p_2}{T_2} \Delta P_2} = \left(1 + \frac{f}{a_1}\right) E_1 A_1 \phi'^1 \sqrt{\frac{p_1}{T_1} \Delta P_1} \quad (2)$$

The unknown variable, total temperature  $T_2$ , may now be obtained from the equation

$$T_2 = \left[ \frac{E_2}{E_1} \frac{A_2}{A_1} \frac{\phi'^2}{\phi'^1} \frac{1}{\left(1 + \frac{f}{a_1}\right)} \right]^2 \frac{p_2}{p_1} \frac{\Delta P_2}{\Delta P_1} T_1 \quad (3)$$

Equation (3) offers a means of determining total temperature  $T_2$  and may be simplified for control application by using the following assumptions:

1. The area ratio  $A_2/A_1$  is a known constant and may be obtained by direct measurement; or by use of equation (3), it may be found by calibrating the system without heating the gas.

2. In a particular design, the operating range of temperatures and of pressure ratios is comparatively small and the conversion factor  $\phi$  may be selected as constant. The factor  $\phi$  is a multiplication factor by which the hydraulic equation is converted to the compressible-flow equation. The equation for the conversion factor  $\phi$  therefore varies with each combination of static pressure  $p$  or total pressure  $P$  and static temperature  $t$  or total temperature  $T$  used in the hydraulic equation so that the compressible-flow equation is always obtained. The expression for determining the actual value of the conversion factor  $\phi'$  when  $\rho = \frac{p}{RT}$  is developed in appendix B for the case in which the density  $\rho$  is proportional to the static pressure  $p$  divided by the total temperature  $T$ . This density relation is desirable for this analysis because it incorporates the total rather than the static temperature and also because the numerical value of the conversion factor  $\phi'$  for each pressure ratio and value of the ratio of specific heats  $\gamma$ , where  $\gamma$  is a function of tempera-

ture, is more nearly equal to 1 than for any other relation of the density  $\rho$ . The deviation of the conversion factor  $\phi'$  from 1 is greatest at critical flow (fig. 1) and increases with increasing temperature because of the change in the ratio of specific heats  $\gamma$ . At the point of critical flow and a value of  $\gamma$  of 1.3, which corresponds to an air temperature of about

$3000^\circ$  R, the conversion factor  $\phi'$  is approximately 0.945. The curve for  $\gamma=1.4$ , which represents an air temperature of about  $500^\circ$  R, is also shown.

3. The area multiplier for thermal expansion  $E_1$  has a negligible change because relatively little change in temperature occurs at station 1 over the range of engine operating conditions.

4. The area multiplier for thermal expansion  $E_2$  increases to a value slightly above 1 with an increase in the fuel-air ratio  $f/a_1$  and the ratio  $\frac{E_2}{1 + \frac{f}{a_1}}$  remains a constant. For ex-

ample, with an assumed burner efficiency of 50 percent, the value of  $\frac{E_2}{1 + \frac{f}{a_1}}$  decreases from 0.995 to 0.990 with a temperature differential increasing from  $400^\circ$  to  $1600^\circ$  F between stations 1 and 2.

If the factors  $A_2/A_1$ ,  $\phi'^2/\phi'^1$ ,  $E_1$ , and  $\frac{E_2}{1 + \frac{f}{a_1}}$  are combined into a single constant  $K$ , equation (3) becomes

$$T_2 = K^2 \frac{p_2}{p_1} \frac{\Delta P_2}{\Delta P_1} T_1 \quad (4)$$

This equation may be adapted for control application. The average values of velocity pressures  $\Delta P_1$  and  $\Delta P_2$  may be obtained by using single pitot-static tubes at each station in a gas stream of known velocity distribution.

Equation (4) may be further simplified for the case in which the ratio  $p_2/p_1$  may be considered constant. Although an investigation should be made into the feasibility of making this assumption for any particular application, it will serve to simplify equation (4) for illustrating a unique hydraulic control system subsequently explained. For this condition, combination of the constants  $K^2$  and  $p_2/p_1$  into a single constant  $K'$  and substitution into equation (4) gives

$$T_2 = K' \frac{\Delta P_2}{\Delta P_1} T_1 \quad (5)$$

#### CONTROL EQUATION FOR CONDITION OF CRITICAL FLOW

Another simplification of temperature equation (3) may be made when critical flow exists at station 2 after the gas is heated. Equation (3) may be rewritten as

$$T_2 = \left[ \frac{E_2}{E_1} \frac{A_2}{A_1} \frac{1}{\phi'^1 \left(1 + \frac{f}{a_1}\right)} \right]^2 (\phi'^2)^2 \frac{p_2}{p_1} \frac{P_2 - p_2}{\Delta P_1} T_1 \quad (6)$$

where  $P_2 - p_2$  replaces its equivalent  $\Delta P_2$ , and  $\phi'^2$  is factored out of the bracketed term.

For the condition of critical flow, the pressure ratio  $p_2/P_2$  is

$$\frac{p_2}{P_2} = \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad (7)$$

This equation may be substituted in equation (B8) (appendix B) for its equivalent  $r$  and an expression for  $(\phi'^2)^2$  in terms of  $\gamma$  is obtained

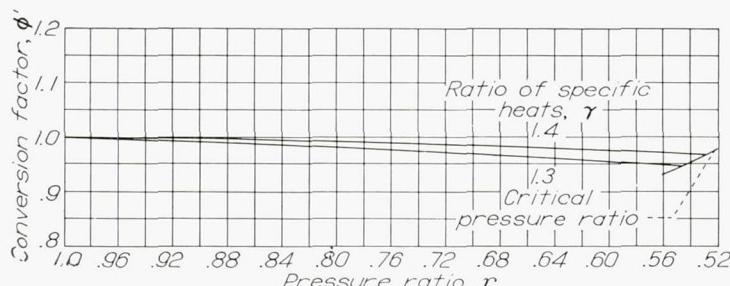


FIGURE 1.—Conversion factor of hydraulic equation to compressible air-flow equation.

$$\phi' = \sqrt{\frac{\frac{1}{r^\gamma} \left( \frac{1-\gamma}{r^\gamma - 1} \right) \gamma}{(1-r)(\gamma-1)}}; r = \frac{\text{static pressure}}{\text{total pressure}}.$$

ture, is more nearly equal to 1 than for any other relation of the density  $\rho$ . The deviation of the conversion factor  $\phi'$  from 1 is greatest at critical flow (fig. 1) and increases with increasing temperature because of the change in the ratio of specific heats  $\gamma$ . At the point of critical flow and a value of  $\gamma$  of 1.3, which corresponds to an air temperature of about

$$\frac{(\phi'_2)^2}{1 - \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}} = \frac{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}{2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}} \quad (8)$$

The terms in the factor  $\left[\frac{F_2 A_2}{E_1 A_1} \frac{1}{\phi'_1 (1+f/a_1)}\right]$  in equation (6)

are assumed constant in the manner previously discussed for determining the constant  $K$  in equation (4) and are designated by the constant  $B$ . Substituting  $B$  and the equivalent of  $p_2$  from equation (7) and  $(\phi'_2)^2$  from equation (8) causes equation (6) to become

$$T_2 = B^2 \frac{\frac{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}{2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}} \frac{P_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}}{p_1} \frac{P_2 - P_1 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}}{\Delta P_1} T_1}{1 - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}}$$

This equation simplifies to

$$T_2 = B^2 \frac{\gamma}{2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{P_2^2}{p_1 \Delta P_1} T_1 \quad (9)$$

The factor  $\frac{\gamma}{2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}$  varies about 5 percent with a change in temperature from  $500^\circ$  to  $3000^\circ$  R. In the case of a temperature-limiting control for which the value of total temperature  $T_2$  is predetermined, the value of this factor may also be predetermined. A constant  $B'$  may then be used to replace the term  $\frac{T_2}{B^2 \frac{\gamma}{2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$  for a par-

ticular value of  $T_2$  and equation (9) may be expressed as

$$B' = \frac{P_2^2 T_1}{p_1 \Delta P_1} \quad (10)$$

In the operation of current gas-turbine engines in which the ratio  $P_2/P_1$  remains substantially constant, equation (10) may be written as

$$B'' = \frac{P_1^2 T_1}{p_1 \Delta P_1} \quad (11)$$

where  $B''$  is a constant. This control equation is unusual because sensing elements are required only before the combustion zone.

The application of a control operating in accordance with equation (10) or (11) can be illustrated by reference to the steady-state operating curve of a typical turbojet engine (fig. 2). The control operating line is shown superimposed on this engine-operating curve. In this engine, critical flow exists in the turbine nozzle box for engine speeds above 8000 rpm. For speeds less than this value, the nozzle-box Mach number  $M$  is less than 1. The maximum operating

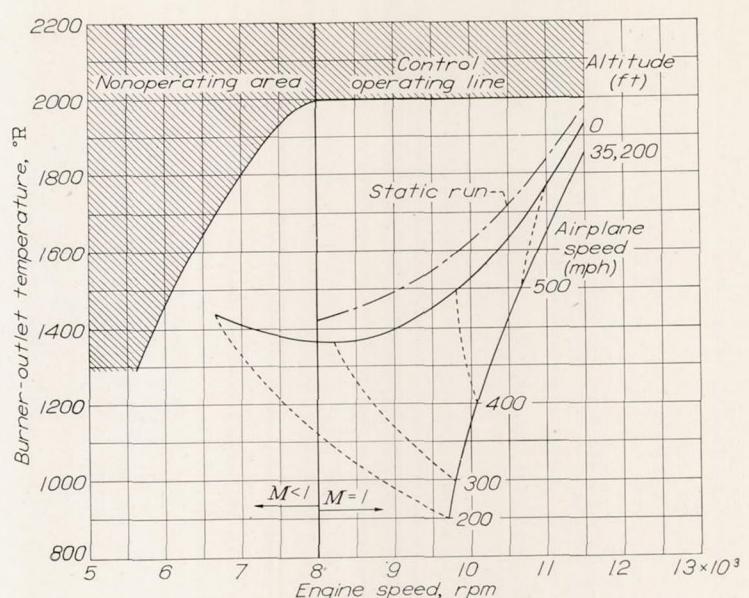


FIGURE 2.—Performance of current turbojet engine showing temperature-limit operating curve. Engine idling speed, approximately 4000 rpm; landing specifications require minimum speed of 8000 rpm. Nozzle-box Mach number,  $M$ .

temperature shown,  $2000^\circ$  R, is the maximum permissible gas temperature specified for this particular engine. Below an engine speed of 8000 rpm, the temperature allowed by the control decreases with decreasing engine speeds; the values shown in figure 2 were determined from equation (10) and from typical engine data. The difference between the steady-state temperature at any engine speed and altitude and the corresponding temperature allowed by the control is the temperature differential available for acceleration. The maximum permissible gas temperature is allowed by the control only for those conditions of engine operation for which critical flow exists. Most normal operation occurs at engine speeds above 8000 rpm where the control allows operation at the maximum permissible temperature. From the shape of the operating curve, the temperatures required for steady-state operation at low engine speeds apparently would not be allowed by the control. Making the control inoperative below some speed that must be determined from engine data therefore becomes necessary.

#### APPLICATION OF CONTROL EQUATIONS

A system designed to control total temperature  $T_2$  by action based on the values of the other variables of equation (5) is schematically shown in figure 3. Equation (5) was selected for illustration because a simpler presentation can be made than with the more basic and more generally applicable equation (4). This control system is unusual in that the required multiplication and division are accomplished by hydraulic means. Temperature control is maintained by the automatic regulation of resistance valve A in the fuel line, which regulates the fuel flow to the nozzles to limit the gas temperature to the predetermined maximum value. Valves B, C, and D can be located in any convenient fluid-flow line through which continuous flow is maintained, such as a fuel-bypass line or a lubricating-oil line. The following relation is obtained by equating the mass rate of flow through valves C and D:

$$C_C A_C \sqrt{\Delta p_C} = C_D A_D \sqrt{\Delta p_D}$$

or

$$C_C^2 A_C^2 = C_D^2 A_D^2 \frac{\Delta p_D}{\Delta p_C} \quad (12)$$

The temperature at station 1 so controls the valve area  $A_D$  that the following relation is maintained:

$$A_D^2 = G T_1$$

where  $G$  is a constant.

The velocity pressure  $\Delta P_1$  is applied to diaphragm-operated valve B, which maintains the pressure drop across valve C,  $\Delta p_C$ , equal to  $\Delta P_1$ . The velocity pressure  $\Delta P_2$  is applied across diaphragm-operated valve A, which controls the fuel flow to the nozzles and thereby controls the magnitude of  $\Delta P_2$ . Valve A is in balance when the pressure drop across valve D,  $\Delta p_D$ , equals  $\Delta P_2$ . Substitutions may therefore be made in equation (12) for  $A_D^2$ ,  $\Delta p_D$ , and  $\Delta p_C$ :

$$A_C^2 = G \frac{\Delta P_2}{\Delta P_1} T_1 \left( \frac{C_D}{C_C} \right)^2 \quad (13)$$

When equation (13) is compared with equation (5), total temperature  $T_2$  is found to equal  $\frac{K_1}{G} \left( \frac{C_C}{C_D} \right)^2 A_C^2$ . Area  $A_C$

may therefore be set to give a predetermined value of total temperature  $T_2$ . If the actual value of total temperature  $T_2$  is less than this value, the value of velocity pressure  $\Delta P_2$  is less than static-pressure difference  $\Delta p_D$  and valve A tends to open, allowing more fuel to enter the nozzles. If the actual value of total temperature  $T_2$  is greater than this predetermined value, the opposite action occurs. If there are no other controls in the fuel-supply line to the nozzles, the control tends to maintain the total temperature  $T_2$  as determined by area  $A_C$ . If, however, controls exist in the system that restrict fuel flow in accordance with the demands of requirements other than maintenance of a maximum gas temperature, this control will act only to limit the maximum value of total temperature  $T_2$  to the value determined by the setting of area  $A_C$ .

A modification of this control is shown in figure 4. The operation of this second control is essentially the same as that of the first (fig. 3) except that a motion that is linear with changes in gas temperature is obtained by the addition of a servosystem. This temperature-dependent motion may be utilized in a control system that uses temperature as a control parameter in conjunction with other engine parameters for power-plant operation. Equations (11) and (12) are still valid, the only difference between the systems being in the method of control. The static-pressure drop across valve C,  $\Delta p_C$ , is controlled by a diaphragm-operated servo-valve and a servopiston to maintain static-pressure drop  $\Delta p_C$  equal to the pressure difference  $\Delta P_1$ . Valve B is a diaphragm-operated throttling valve, which acts to maintain  $\Delta p_D$  equal to  $\Delta P_2$ . The manual lever controls a servo-valve and a servopiston-regulating valve A to maintain a set total temperature  $T_2$ . Because area  $A_C$  is a measure of total temperature  $T_2$ , the position of the stem of valve C is a measure of temperature as indicated by the temperature indicator. The manual lever may be so set that the servo-

valve and the piston-positioning valve A determine the equilibrium position of the stem of valve C, which, in effect, determines the total temperature  $T_2$ .

Valves B, C, and D (fig. 4) in the bypass line are continuously functioning and the position of valve C is a con-

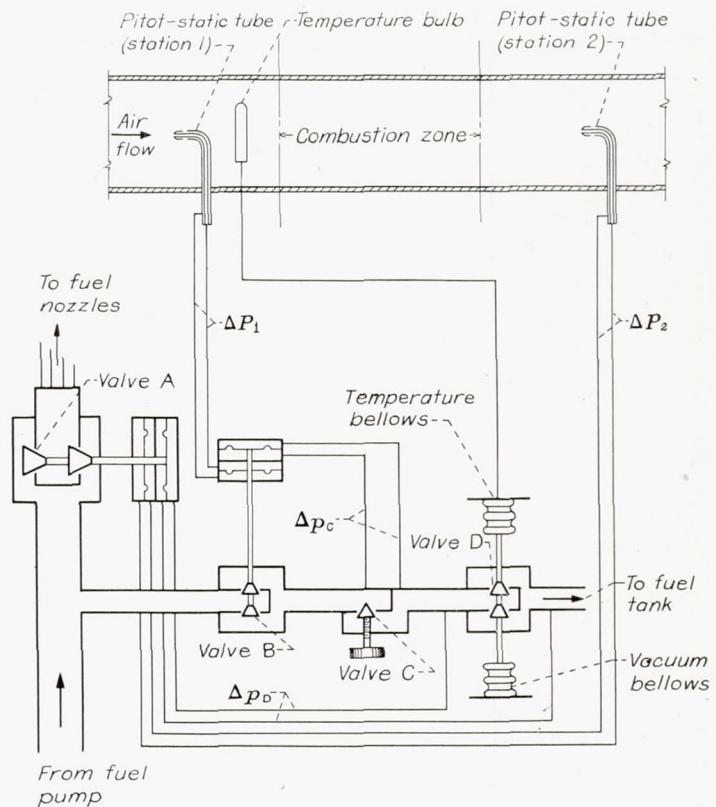


FIGURE 3.—Schematic diagram of basic temperature control.

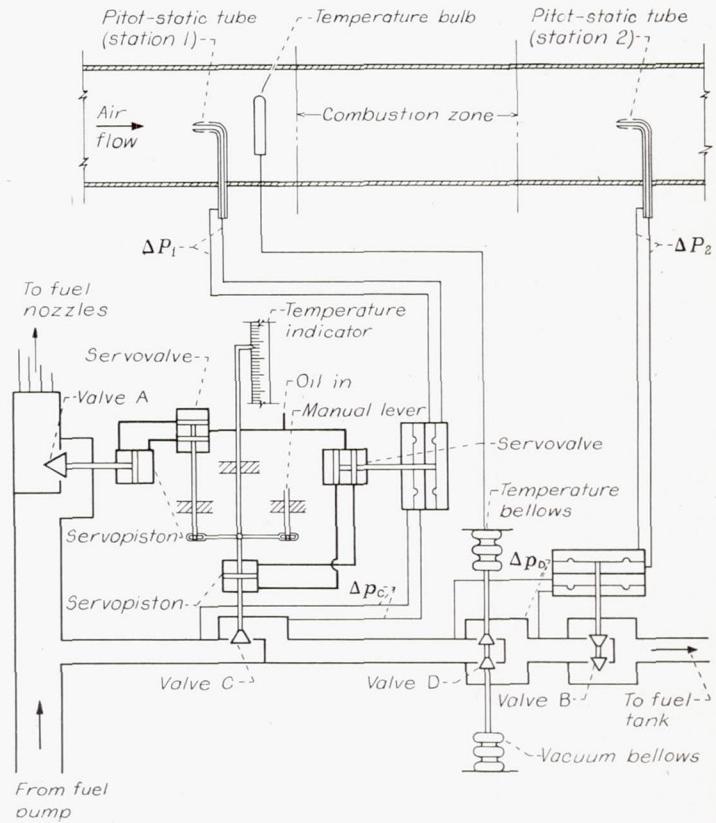


FIGURE 4.—Schematic diagram of temperature control with automatic temperature indication.

tinuous indication of temperature. The temperature indicator therefore operates when the control is used either as a temperature-limiting control or as a regulating control. This control system differs from the system shown in figure 3 in that the valve area  $A_C$  (fig. 3) is manually set by the pilot or may be preset to maintain a particular limiting temperature. This area setting indicates temperature only when the control is used as a regulating control; otherwise, the area setting is that of the limiting temperature.

A control operating according to the control equations based on critical flow at one station is schematically shown in figure 5. Either equation (10) or (11) may be used as the temperature-control equation, depending on whether total

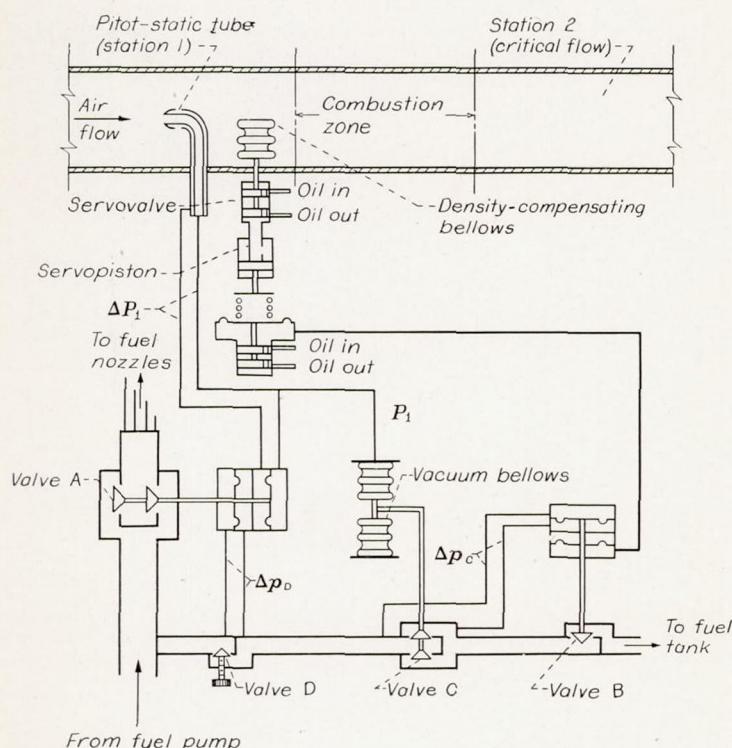


FIGURE 5.—Schematic diagram of temperature control based on critical-flow equation.

pressure  $P_2$  or  $P_1$  is to be used to actuate the control. The schematic diagram in figure 5 is based on equation (11). This control is similar to those diagrammed in figures 3 and 4 in that valve A controls the fuel flow to the engine, and that valves B, C, and D are located in a convenient fuel-flow line. The density term  $p_1/T_1$  in equation (11) is so applied to a density-compensating system that valve B controls the pressure drop  $\Delta p_C$  proportionally to  $T_1/p_1$ . The area  $A_C$  is made proportional to the total pressure  $P_1$  and the area  $A_D$  is made proportional to the constant  $B_2$  of equation (11), which depends on the value of total temperature  $T_2$  selected. The static-pressure difference  $\Delta p_D$  then so controls valve A that  $\Delta P_1$  equals  $\Delta p_D$ . When these substitutions are made, control equation (12) is seen to operate according to temperature equation (11).

## CONCLUSIONS

The analysis indicates that determination of combustion-gas temperatures from the thermodynamic relations involving gas temperature before combustion and gas pressures before and after combustion appears practical for high-temperature application, such as gas-turbine temperature control. For critical flow in the turbine nozzles, the analysis indicates that combustion-gas temperature can be determined and controlled from measurements taken only upstream of the combustion zone.

FLIGHT PROPULSION RESEARCH LABORATORY,  
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,  
CLEVELAND, OHIO, March 4, 1948.

## APPENDIX A

### SYMBOLS

The following symbols were used in the report:

$A$	area, sq ft
$a$	air or gas flow, lb/sec
$B, G, K$	constants
$C$	coefficient of discharge
$E$	area multiplier for thermal expansion of duct
$f$	fuel flow, lb/sec
$g$	acceleration due to gravity, ft/sec <sup>2</sup>
$M$	Mach number
$P$	total pressure, lb/sq ft absolute
$\Delta P$	total pressure minus static pressure, $P-p$
$p$	static pressure, lb/sq ft absolute
$\Delta p$	static-pressure drop across control valves
$R$	gas constant
$r$	ratio of static pressure to total pressure, $p/P$
$T$	total temperature, °R

$t$	static temperature, °R
$V$	velocity, ft/sec
$\gamma$	ratio of specific heats at constant pressure and constant volume
$\rho$	arbitrarily selected density, lb/cu ft
$\rho_0$	free-stream density, lb/cu ft
$\rho_s$	stagnation density, lb/cu ft
$\phi$	conversion factor of hydraulic equation to compressible-flow equation
$\phi'$	particular value of $\phi$ when $\rho = \frac{p}{RT}$
Subscripts:	
$A, B, C, D$	valves A, B, C, and D in control systems
1	station 1 (before heating)
2	station 2 (after heating)

## APPENDIX B

### COMPRESSIBLE-FLOW EQUATIONS

The hydraulic equation for incompressible flow may be multiplied by an appropriate conversion factor  $\phi$  to obtain the exact equation for compressible flow. This expression for the conversion factor  $\phi$  may be derived from the compressible-flow equation by factoring out the hydraulic equation so that the remaining factor is the expression for the conversion factor  $\phi$ .

Bernoulli's theorem for compressible flow may be written as

$$V = \left[ \frac{2g\gamma}{\gamma-1} \left( \frac{P}{\rho_t} - \frac{p}{\rho_0} \right) \right]^{\frac{1}{2}} \quad (\text{B1})$$

The weight-flow rate is

$$a = A\rho_0 V \quad (\text{B2})$$

Substituting equation (B1) in equation (B2) and replacing stagnation density  $\rho_t$  with the equivalent adiabatic relation

$$\rho_0 \left( \frac{P}{p} \right)^{\frac{1}{\gamma}} \text{ gives}$$

$$a = A \left\{ 2g \left( \frac{\gamma}{\gamma-1} \right) \rho_0 \left[ \frac{P}{\left( \frac{P}{p} \right)^{\frac{1}{\gamma}}} - p \right] \right\}^{\frac{1}{2}}$$

The free-stream density  $\rho_0$  may be replaced by its equivalent

$\frac{p}{Rt}$  and the equation simplified:

$$\begin{aligned} a &= A \left\{ 2g \left( \frac{\gamma}{\gamma-1} \right) \frac{p}{Rt} \left[ \frac{P^{\left( \frac{1-\frac{1}{\gamma}}{\gamma} \right)} - p^{\left( \frac{1-\frac{1}{\gamma}}{\gamma} \right)}}{p^{\frac{1}{\gamma}}} \right] \right\}^{\frac{1}{2}} \\ a &= A \left\{ 2g \left( \frac{\gamma}{\gamma-1} \right) \frac{p^2}{Rt} \left[ \frac{P^{\left( \frac{1-\frac{1}{\gamma}}{\gamma} \right)} - p^{\left( \frac{1-\frac{1}{\gamma}}{\gamma} \right)}}{p^{\left( \frac{1-\frac{1}{\gamma}}{\gamma} \right)}} \right] \right\}^{\frac{1}{2}} \\ a &= A \left\{ 2g \left( \frac{\gamma}{\gamma-1} \right) \frac{p^2}{Rt} \left[ \left( \frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \end{aligned} \quad (\text{B3})$$

This expression for compressible flow may be written as

$$a = A\phi \sqrt{2g\rho(P-p)} \quad (\text{B4})$$

where

$$\phi = \left\{ \frac{1}{\rho(P-p)} \left( \frac{\gamma}{\gamma-1} \right) \frac{p^2}{Rt} \left[ \left( \frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \quad (\text{B5})$$

This expression of  $\phi$  involves the density  $\rho$ , which also appears in equation (B4) and may be arbitrarily selected as a ratio involving total pressure  $P$  or static pressure  $p$  divided by total temperature  $T$  or static temperature  $t$ . For the case in which the density  $\rho$  is selected as  $p/RT$ , the conversion factor  $\phi$  is designated as  $\phi'$  and equation (B5) is simplified as follows:

$$(\phi')^2 = \frac{Tp\gamma \left[ \left( \frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{t(P-p)(\gamma-1)} \quad (\text{B6})$$

The adiabatic relation of the temperatures is

$$\frac{T}{t} = \left( \frac{p}{P} \right)^{\frac{1-\gamma}{\gamma}} \quad (\text{B7})$$

This relation may be substituted into equation (B6) to obtain

$$(\phi')^2 = \frac{\left( \frac{p}{P} \right)^{\frac{1-\gamma}{\gamma}} \gamma \left[ \left( \frac{P}{p} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\left( \frac{P-p}{p} \right) (\gamma-1)}$$

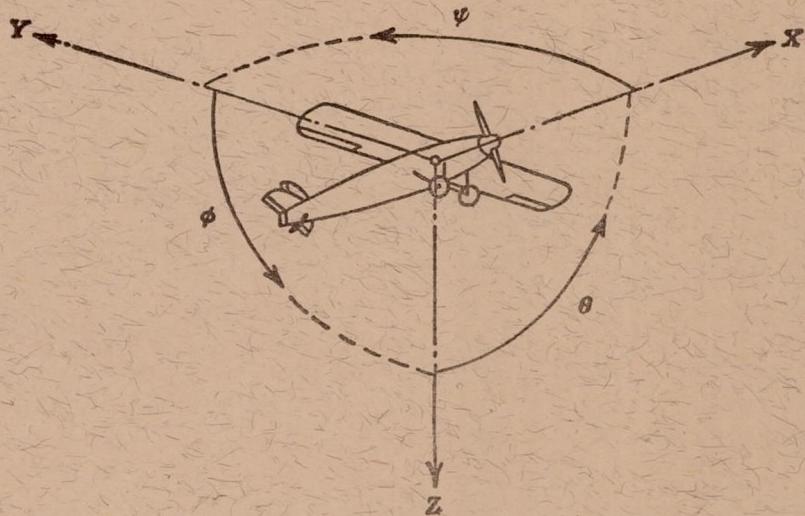
If the pressure ratio  $p/P$  is set equal to  $r$  this equation becomes

$$(\phi')^2 = \frac{\left( r^{\frac{1-\gamma}{\gamma}} \right) \left( r^{\frac{1-\gamma}{\gamma}} - 1 \right) \gamma}{\left( \frac{1}{r} - 1 \right) (\gamma-1)}$$

or

$$(\phi')^2 = \frac{r^{\frac{1}{\gamma}} \left( r^{\frac{1-\gamma}{\gamma}} - 1 \right) \gamma}{(1-r)(\gamma-1)} \quad (\text{B8})$$

A plot of the conversion factor  $\phi'$  against the pressure ratio  $r$  is presented in figure 1, which shows the error that may be expected from neglecting  $\phi$  in the hydraulic equation (B4) where the density  $\rho$  is  $p/RT$ . The greatest deviation of the conversion factor  $\phi'$  from 1 occurs at the critical pressure ratio, at which  $\phi'$  is approximately 0.945 for the ratio of specific heats  $\gamma$  equal to 1.3 for air at a temperature of 3000° R.



Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \rightarrow Z$	Roll	$\phi$	u	p
Lateral	Y	Y	Pitching	M	$Z \rightarrow X$	Pitch	$\theta$	v	q
Normal	Z	Z	Yawing	N	$X \rightarrow Y$	Yaw	$\psi$	w	r

### Absolute coefficients of moment

$$C_i = \frac{L}{qbS} \quad C_m = \frac{M}{qcS} \quad C_n = \frac{N}{qbS}$$

(rolling)      (pitching)      (yawing)

Angle of set of control surface (relative to neutral position),  $\delta$ . (Indicate surface by proper subscript.)

#### 4. PROPELLER SYMBOLS

$D$	Diameter
$p$	Geometric pitch
$p/D$	Pitch ratio
$V'$	Inflow velocity
$V_s$	Slipstream velocity
$T$	Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$
$Q$	Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$

$P$	Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$
$C_s$	Speed-power coefficient $= \sqrt{\frac{\rho V^5}{P n^2}}$
$\eta$	Efficiency
$n$	Revolutions per second, rps
$\Phi$	Effective helix angle $= \tan^{-1} \left( \frac{V}{\frac{2 \pi n D}{3}} \right)$

## 5. NUMERICAL RELATIONS

$$1 \text{ hp} = 76.04 \text{ kg-m/s} = 550 \text{ ft-lb/sec}$$

1 metric horsepower = 0.9863 hp

1 mph = 0.4470 mps

1 mph = 0.447 mps

$$1 \text{ lb} = 0.4536 \text{ kg}$$

$$1 \text{ kg} = 2.2046 \text{ lb}$$

1 mi = 1,609.35 m = 5,280 ft

1 m = 3.2808 ft